Foreign Exchange Market Inefficiency and Exchange Rate Anomalies

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Abstract

This paper develops a non-linear Uncovered Interest Parity, UIP, framework with fx market inefficiency that nests the non-linear econometric UIP models and the “infrequent portfolio adjustment” model of Bacchetta and van Wincoop. FX market inefficiency means that for any nonzero home minus foreign nominal interest differential, ID, there is always unexploited expected profit, which creates a tendency for a negative value for Fama’s beta coefficient. However, as ID decays over time, this tends to generate a positive value for Fama’s beta. The sign of beta is uncertain, depending on the relative values for the degree of fx market inefficiency and the rate of decay in ID. Simulations imply that the model is consistent with many exchange rate anomalies.

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JEL # F31, F37
1. INTRODUCTION

According to the concept of Uncovered Interest Parity, UIP, in an efficient fx market the currency of a high interest rate country will appreciate in the period when its interest rate increases, and depreciate in future periods. The initial appreciation brings about ex ante UIP and the ensuing depreciation generates ex post UIP. The UIP puzzle (or forward bias puzzle) is that the currency of a high interest rate country often appreciates in the period immediately after its interest rate increases. The empirical tests on this subject obtain estimates of the relationship between the home minus foreign nominal interest rate differential, ID, and the percentage change in the spot rate one period later. This relationship is embodied in a regression coefficient that is commonly called Fama’s $\beta^1$. The latter should be plus unity if UIP holds.

Scholarly studies find that: (a) beta is often negative with an average value close to minus unity, i.e., the UIP puzzle; $^2$ (b) estimates of beta are extremely variable over time, sometimes going from negative to positive for the same country; (c) estimates of beta are often positive when only outlier values for ID are used (i.e., there is “extreme support” for UIP), but are negative when only smaller (non-outlier) IDs are used; and (d) the currency of the high interest rate

$^1$ See Fama (1984), and more recently Baillie and Bollerslev (2000), Gourinchas and Tornell (2004), Chaboud and Wright (2005), Chinn (2006), Sarno, et. al. (2006), and Burnside, et. al (2009).

$^2$ Froot and Thaler (1990) report an average value for $\beta$ of -0.88 in more than 70 Fama type regressions.
country tends to appreciate for many periods following an increase in its interest rate, but it depreciates eventually; that is, “delayed overshooting” occurs\(^3\).

Froot and Thaler (1990) suggest several possible reasons for the UIP puzzle, one of which is the possibility that the fx market is sometimes inefficient. More recently, Baillie and Killic (2006), Sarnos, et al. (2006), and Baillie and Chang (2011) have estimated non-linear econometric UIP models that are consistent with market inefficiency. They find that Fama’s \(\beta\) is positive when sufficient profit opportunities exist. On the other hand, they conclude that Fama’s \(\beta\) is negative when expected returns are low, as, for example, when interest rate differentials are not large. One key toward solving the UIP puzzle might be to explain why \(\beta\) is negative for relatively smaller values for ID, when, presumably, no speculation takes place. Sarnos, et al. (2006) point out that we do not know the answer to this question.

Another attempt to model fx market inefficiency is that of Bacchetta and van Wincoop (2010). They rigorously develop and simulate an overlapping generations model wherein investors decide on the optimum combination of home versus foreign assets only once in their lifetime. With reasonable parameter values, simulations of their model generate negative values for Fama’s \(\beta\) coefficient. However, in contrast to the findings of the non-linear models, such estimates are always negative.

\(^3\) See Eichenbaum and Evans (1995). In Dornbusch (1976), an exogenous change in interest rates induces the spot rate to change instantaneously by more than any change in its expected long run equilibrium value. Then the exchange rate begins to return back toward its long run value in the very next time period. Delayed overshooting occurs if the spot rate begins to converge back toward its long run equilibrium value only after many time periods.
The objective of this paper is to synthesize and generalize the important contributions in the non-linear approach and the infrequent portfolio adjustment approach. This is done in a straight-forward manner by assuming that the spot rate might not adjust enough within any one time period to bring about ex ante UIP. No attempt is made to model why this is true, but the discussion follows the lead of Sarno, et. al. (2006) by using Lyons (2001) “limits to speculation” hypothesis. However, the model is consistent with any reason for fx market inefficiency.\(^4\)

Market inefficiency is modeled via the assumption that in any one time period the speculative movement of funds eliminates only \(\lambda\%\) of the expected profit from carry-trade, with \(0 < \lambda \leq 1\). Furthermore, it is assumed that \(\lambda\) is positively correlated with the absolute value for ID; that is, the fx market becomes progressively more inefficient as ID decreases in absolute value. This is consistent with Lyons (2001) “limits to speculation” hypothesis, and with the nonlinear models of Baillie and Killic (2006), Sarnos, et al. (2006), and Baillie and Chang (2011).

When the home, USA, interest rate increases in period \(t\) relative to the foreign rate, then ID\((t)\) becomes positive, and funds flow from fx to dollars, thereby appreciating the dollar in period \(t\). However, the spot rate does not move enough in period \(t\) to satisfy ex ante UIP; this means that unexploited profit exists at the end of period \(t\). The existence of unexploited profit creates a tendency for funds to move again from fx to dollars in the next time period, \(t+1\). This, in itself,

serves to appreciate the dollar in $t+1$. This is called the “unexploited profit effect”.

However, all nonzero IDs tend to decay slowly over time. If ID decays somewhat in period $t+1$, this tends to depreciate the dollar in $t+1$, which is called the “decaying ID effect”. The net effect on the spot rate in period $t+1$ is uncertain, depending on the relative strengths of the opposing forces. If the “unexploited profit effect” dominates, then the dollar appreciates again in period $t+1$, which is consistent with the UIP puzzle. Within our model, this is more likely for smaller absolute values for ID. Thus, the model is consistent with the fact that smaller IDs have negative betas in the “extreme support” literature. If the “decaying ID effect” dominates in period $t+1$, then the dollar depreciates in $t+1$, and this implies that Fama’s $\beta$ is positive. This is more likely for large absolute values for ID, as in the “extreme support” literature.

Estimates of Fama’s $\beta$ will vary over different time intervals if the relative frequency of large versus smaller IDs varies. Finally, if the “unexploited profit effect” dominates initially (so that the UIP puzzle exists) then the dollar can continue to appreciate for many periods. However, the strength of the “unexploited profit effect” gradually approaches zero. Therefore, eventually the dollar is certain to start depreciating, as in “delayed overshooting”.

2. A UIP FRAMEWORK WITH MARKET INEFFICIENCY

2.1 Notation and Assumptions

The model is similar to what appears in Gourinchas and Tornell (2004) and Craighead et al. (2010), except for the assumption of fx market inefficiency.
It is extremely simple in order to focus on the key ideas without unnecessary complications. The discussion assumes that the home country is the USA and that its interest rate exceeds the interest rate abroad, i.e., ID is positive. As pointed out above, ID(t) represents the home minus foreign nominal interest rate differential at time t. Agents always know the current value for ID(t) and they form expectations with respect to future values for ID. Nonzero interest differentials decay over time at an average rate of $\Phi$ per period, but the actual rate of decay between any two periods t and (t+1), denoted by $\phi(t+1)$, need not equal its average value. Agents know these facts and their expectations about the rate of decay for any nonzero ID(t), denoted by $\phi^*$, satisfy rational expectations; that is, $\phi^* \equiv \Phi$.$^5$

The model also assumes that there are no transactions costs and that agents are risk neutral. This means that carry-trade can occur any time that ID is nonzero, and that speculators have an infinitely long time horizon, “n”. The log of the spot rate, $S(t) = \$/fx$, is represented by s(t). $s^*$ is the log of the correctly anticipated long run equilibrium value for s, which is fixed exogenously. At time t the correctly anticipated cumulative percentage change in the spot rate is given by $[s^* – s(t)]$; if this is positive then the dollar is expected to depreciate.

For any given nonzero ID(t), the anticipated cumulative net interest from carry-trade is given by $\Omega(t)$ as defined by (1a). Since, by assumption, investors always expect that ID will decay at the known average rate, $\Phi$, it follows that $\Omega(t)$ reduces to the expression in (1b). At the beginning of any period t, the expected cumulative profit, $E[\Pi(t)]$, from borrowing fx and investing in dollar assets equals

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$^5$ This differs from Gourinchas and Tornell who assume that investors make systematic errors when predicting interest rate differentials.
the anticipated cumulative net interest, \( \Omega(t) \), minus any expected (as of the end of the previous period) cumulative future depreciation of the dollar, \([s^* - s(t-1)]\), as in (1c).

\[
\Omega(t) = ID(t) + E[ID(t+1)] + E[ID(t+2)] + \ldots E[ID(t+n)] \\
\Omega(t) = ID(t) + (1-\phi^*)ID(t) + \ldots(1-\phi^*)^nID(t) = (1/\phi^*)ID(t) = (1/\Phi)ID(t) \\
E[\Pi(t)] = \Omega(t) - [s^* - s(t-1)]
\] (1a) (1b) (1c)

When \( E[\Pi(t)] \geq 0 \), then carry-trade moves funds from fx into dollars and this appreciates the dollar in period \( t \). If \( E[\Pi(t)] \) is negative then funds move out of dollars, thereby depreciating the dollar. The key assumption is that the spot rate usually does not vary in period \( t \) by enough to satisfy ex ante UIP. That is, if \( ID \) goes unexpectedly from zero to positive, then with \( s^* \) fixed, \( s(t) \) decreases by enough to eliminate only \( \lambda \% \) of \( E[\Pi(t)] \), where \( 0 < \lambda \leq 1 \). The degree of fx market inefficiency is \( (1 - \lambda) \). In a perfectly efficient market \( \lambda = 1 \) and there is a zero degree of inefficiency.⁶

The nonlinear UIP frameworks in Baillie and Killic (2006), Sarnos, et al. (2006), and Baillie and Chang (2011) assume that the fx market more closely

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⁶ The model in Bacchetta and van Wincoop (2010) implicitly assumes that \( \lambda < 1 \), because infrequent portfolio adjustments alone will not ensure a negative value for Fama’s \( \beta \) if each group of investors has enough funds to bring about ex ante UIP.
approaches ex ante UIP as the profitability from speculation increases. Within the
text of our model, this implies that $\lambda$ is endogenous and moves positively with
perceived profit. Sarnos et al. use Sharpe ratios as a proxy for anticipated excess
returns; Baillie and Chang use interest rate differentials, and exchange rate
volatility; finally, among other things, Baillie and Killic use the value for the
forward premium or discount.\textsuperscript{7} The approach here is given by (2) which assumes
that $\lambda$ increases as the absolute value for ID increases.

$$\lambda(t+k) = \lambda[ \mid ID(t+k) \mid ] \ldots \lambda' > 0$$ (2)

2.2 The Model

The model generates a solution for the percentage change in the spot rate
for any period (t+k) following an exogenous change in the interest rate differential
in period t. There is an implied value for Fama’s beta in each period. Assume,
first, that in period t-1 the spot rate equals its long run equilibrium value, the
interest rate differential is zero, and ID is expected to be zero in all future periods,
as in (3a). In this situation, both $\Omega(t-1)$ and $E[\Pi(t-1)]$ are also zero.

Then let the US interest rate increase unexpectedly so that ID(t) > 0.

Agents correctly expect ID(t) to decay slowly over time at an average rate of $\Phi$

\textsuperscript{7} Since there is a consensus that covered interest parity holds essentially all of the time, the use of
the forward premium or discount amounts to using the interest rate differential.
percent per period, i.e., $\phi^* \equiv \Phi$. Consequently, the anticipated cumulative net interest from carry-trade is given by $\Omega(t)$ in (3b). As of the beginning of period $t$, the expected cumulative profit from carry-trade in (3c) equals $\Omega(t)$ minus $[s^* - s(t-1)]$, the anticipated cumulative future depreciation of the dollar as of the end of period $t-1$; the latter, however is zero from (3a).

$$s^* - s(t-1) = 0 = ID(t-1) = E[ID(t+k)] \ldots \ldots k = 0 \ldots \infty \quad (3a)$$

$$\Omega(t) = \frac{1}{\Phi} ID(t) \quad (3b)$$

$$E[\Pi(t)] = \Omega(t) - [s^* - s(t-1)] = \Omega(t) \quad (3c)$$

In period $t$, speculators borrow fx, sell it spot for dollars, and buy interest earning dollar assets. The spot purchases of dollars instantaneously appreciate the dollar, $ds(t) < 0$, but not enough to satisfy ex ante UIP. The international flow of funds eliminates only $\lambda(t)$ percent of expected profit in period $t$, as indicated by (4a). Hence, it follows that the anticipated cumulative depreciation of the dollar in future periods is given by (4b). Ex ante UIP does not hold because the anticipated cumulative dollar depreciation is less than anticipated profit; unexploited expected profit is $[1 - \lambda(t)] E[\Pi(t)] = [1 - \lambda(t)] \Omega(t)$.

An appendix derives (4c), an expression for the change in the spot rate in any future period, $t+k$, after the exogenous increase in $ID(t)$. There is a tendency for the dollar to appreciate, $ds(t+k) < 0$, in future periods via the unexploited
profit from earlier periods. Also, there is a tendency for the dollar to depreciate because ID(t) decays over time. Furthermore, the relationship between the change in the spot rate in any period and the one period lagged value for ID (which is embedded in the Ω term) is nonlinear, because λ varies positively with ID.

\[
ds(t) = -\lambda(t) E[\Pi(t)] = -\lambda(t) \Omega(t) < 0 \tag{4a}
\]

\[
s^* - s(t) = +\lambda(t) E[\Pi(t)] = +\lambda(t)\Omega(t) = -ds(t) \tag{4b}
\]

\[
ds(t+k) = \lambda(t+k)[\phi(t+k) - [1-\lambda(t+k)]]\Omega(t+k-1) + \\
\lambda(t+k)[1-\lambda(t+k)] [s^* - s(t+k-2)] \tag{4c}
\]

2.3 Generating Exchange Rate Anomalies

This section shows that the model yields an expression for a beta coefficient for each time period, and it uses this to prove that the model is consistent with the four anomalies listed above. The analysis focuses on the theoretical value for beta one period after the home (USA) interest rate increases exogenously relative to the interest rate abroad. A footnote shows that similar conclusions hold for any time period.

2.3.1 The UIP Puzzle
The \( - \lambda(t+k)[1 - \lambda(t+k)] \Omega(t+k-1) \) term in (4c) tells us that the change in the spot rate in any period will have a tendency to be negative when the home (USA) interest rate is higher than abroad. This is tied up with the fact that there is always some unexploited profit from previous periods that will attract funds into the USA and appreciate the dollar; this is the "unexploited profit effect" defined above. On the other hand, if ID decays in period \( t+k \), then \( \phi(t+k) > 0 \) in (4c), and the "decaying ID effect", as given by \( \lambda(t+k)\phi(t+k)\Omega(t+k-1) \), tends to depreciate the dollar. This happens because a smaller ID implies a decrease in anticipated cumulative net interest from carry trade, which in turn reduces expected profit. Hence, the anticipated cumulative future depreciation of the dollar must be less, which means that the dollar must depreciate somewhat. The net influence of the "unexploited profit effect" and the "decaying ID effect" on \( ds(t+k) \) is uncertain.

In addition, the \([s^* - s(t+k-2)]\) term in (4c) reflects the anticipated cumulative future depreciation of the dollar, if the dollar appreciated in one or more previous periods. As this increases, it reduces expected profit from moving funds from fx into dollars, thereby exerting a positive influence on \( ds(t+k) \), i.e., a downward pressure on the dollar. In period \( t+1 \), the \([s^* - s(t+k-2)]\) term in (4c) becomes \([s^* - s(t-1)]\), which is zero from (3a). Therefore, in period \( t+1 \), the sign of \( ds(t+1) \) in (4c) depends only on the sign of \([\phi(t+1) - [1-\lambda(t+1)]\], i.e., on the
relative magnitudes of the most recent *actual* rate of decay in ID, \( \phi(t+1) \), versus
the degree of fx market inefficiency, \([1- \lambda(t+1)]\).

It is useful to rewrite (4c) for period \( t+1 \) in the form of a Fama equation.
Appropriate substitutions yield (5a) with \( \beta(t+1) \) and \( X(t+1) \) given by (5b) and
(5c), respectively. Note that: (a) \( \beta(t+1) \) will not equal +1 even though the fx
market is perfectly efficient unless the most recent actual rate of decay in ID,
\( \phi(t+1) \), equals the average rate of decay, \( \Phi \); and (b) the anticipated cumulative
future depreciation of the dollar represents a here-to-for unrealized missing
variable in a Fama regression. The expected value for \( \beta(t+1) \) is determined
according to (5d). If the fx market were perfectly efficient, then \( E[\beta(t+1)] = +1 \)
in (5d), as in standard thinking about UIP. However, if fx market inefficiency
exists, then on those occasions where \( E[\beta(t+1)] \) is positive, it’s value will be less
than + \( \lambda \), and, thus, perhaps much less than +1.

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8 To obtain a similar expression for any time period \( t+k \), rewrite (4c) as:

\[
ds(t+k) = \lambda(t+k) \left\{ \phi(t+k) - [1 - \lambda(t+k)] \right\} \left(1/\Phi\right) \text{ID}(t+k-1) + \lambda(t+k)[1 - \lambda(t+k)] \left[s^* - s(t+k-2)\right].
\]

Clearly, \( \beta(t+k) = \lambda(t+k) \left\{ \phi(t+k) - [1 - \lambda(t+k)] \right\} \left(1/\Phi\right) \), and \( X(t+k) = \lambda(t+k)[1 - \lambda(t+k)] \left[s^* - s(t+k-2)\right] \). These
are equivalent to (5b) and (5c). The equivalent general expression for the expected value for
\( \beta(t+k) \) then becomes \( E[\beta(t+k)] = \lambda(t+k) \left(\Phi - [1 - \lambda(t+k)]\right) \left(1/\Phi\right) \). Thus, the conclusions reached in
the text with respect to period \( t+1 \) hold, in general for any period \( t+k \).
Bacchetta and van Wincoop (2010) point out that the average quarterly rate of decay in IDs between the USA and leading OECD countries is approximately 0.20. From this it follows from (5d) that \( E[\beta(t+1)] \) is positive (thereby lending some support for UIP) if \( \lambda \) is greater than 0.80, and is negative (as in the UIP puzzle) for \( \lambda < 0.80 \).

\[
ds(t+1) = \beta(t+1) \text{ID}(t) + X(t+1) \tag{5a}
\]
\[
\beta(t+1) = \lambda(t+1)\left(\frac{\phi(t+1)}{\Phi} - [1-\lambda(t+1)](1/\Phi)\right) \tag{5b}
\]
\[
X(t+1) = \lambda(t+k)[1-\lambda(t+k)]E[s^* - s(t-1)] \tag{5c}
\]
\[
E[\beta(t+1)] = \lambda(t+1)\left(1 - [1 - \lambda(t+1)]/\Phi\right) \tag{5d}
\]

### 2.3.2 “Extreme Support” and Unstable Estimates of Beta

As pointed out above, the assumption that fx market inefficiency increases as the absolute value for ID decreases is consistent with the “limits to speculation” hypothesis of Lyons (2001) and with the nonlinear UIP frameworks in Baillie and Killic (2006), Sarnos, et al. (2006), and Baillie and Chang (2011). From (5d) it is clear that for any given average rate of decay in nonzero IDs, the expected value for \( \beta(t+1) \) is more likely to be positive for larger values for \( \lambda(t+1) \). If the latter are associated with relatively large absolute values for ID, then the model is consistent with the findings of the “extreme support” literature.
On the other hand, if smaller absolute values for ID are associated with smaller values for \( \lambda(t+1) \), i.e., the fx market is less efficient, then such IDs are more likely to generate negative values for \( \beta(t+1) \). This, too, is consistent with the “extreme support” literature. Note well, that the model suggests an answer to Sarno’s unanswered question. Smaller absolute values for ID yield negative values for Fama’s \( \beta \) because the “unexploited profit effect” is likely to dominate the “decaying ID effect” for smaller IDs.

If the relative frequency of large versus smaller IDs varies from one time interval to another, then the \( \lambda \) terms in (5d) vary also. The effect on \( E[\beta(t+1)] \) from variations in \( \lambda(t+1) \) is given by (6a). If the parameters are such that \( E[\beta(t+1)] \) is positive, i.e., if \( \{1 - [1 - \lambda(t+1)]/\Phi \} > 0 \), then decreases in \( \lambda \) are certain to decrease the expected value for \( \beta(t+1) \). On the other hand, if \( E[\beta(t+1)] \) is negative (i.e., if the UIP puzzle exists), then the sign of the change in \( E[\beta(t+1)] \) in response to a variation in \( \lambda(t+1) \) is determined by (6b); that is, \( E[\beta(t+1)] \) could rise or fall with a decrease in \( \lambda(t+1) \).\(^9\)

In either case it is clear that the value for beta will vary as the absolute value for ID varies, because this yields different degrees of fx market inefficiency.

\(^9\) The sign in (6b) is more likely to be positive (negative) as the value for \( \lambda \) gets larger (smaller).
Consequently, estimates of Fama’s $\beta$ over different time intervals are apt to be unstable, depending on the relative frequency of larger versus smaller IDs.

$$\delta E[\beta(t+1)]/\delta \lambda(t+1) = \left\{1 - \frac{1 - \lambda(t+1)}{\Phi} \right\} + \frac{\lambda(t+1)}{\Phi}$$  \hspace{1cm} (6a)$$

$$\text{sgn } \delta E[\beta(t+1)]/\delta \lambda(t+1) = \text{sgn } \left[\Phi - 1 + 2\lambda(t+1)\right]$$  \hspace{1cm} (6b)$$

2.3.3 Delayed Overshooting

A negative value for $\{\phi(t+k) - [1 - \lambda(t+k)]\}$ in (4c) is a necessary and sufficient condition for the dollar to appreciate in period $t+1$. However, this is not true in future periods, because $[s^* - s(t+k - 2)]$ in (4c) is zero in period $t+1$, but it will be progressively more positive over time if the dollar continues to appreciate. That is, if the expected cumulative depreciation of the dollar gets progressively larger over time, then this works against the fact that unexploited profit exists from previous periods.

Thus a negative value for $\{\phi(t+k) - [1 - \lambda(t+k)]\}$ in (4c) is necessary but not sufficient for the dollar to appreciate persistently in future periods. The anticipated cumulative excess interest from carry-trade, $\Omega(t+k-1)$ in (4c), gradually approaches zero as ID decays over time. Consequently, the ever increasing anticipated cumulative dollar depreciation eventually dominates the
ever decreasing anticipated cumulative excess interest from carry-trade. When this happens then funds will flow out of dollars, thereby depreciating the dollar. In brief, the model is consistent with “delayed overshooting”, anomaly (d).\textsuperscript{10}

3. SIMULATIONS

This section simulates the model. We first construct a time series for the home minus foreign nominal interest rate differential, ID. As pointed out above, Bacchetta and van Wincoop (2010) assume a rate of decay in ID of 20% per period, which is approximately the mean for quarterly data between the US versus other leading countries. Our constructed time series for ID has 128 data points (the equivalent of 32 years) and it includes a random term with zero mean, and a variance that is comparable to that for actual quarterly ID data.\textsuperscript{11}

3.1. The UIP or Forward Bias Puzzle

The procedure is to insert the constructed time series of values for ID into the model’s equation (4c) in order to obtain a value for the percentage change in the spot rate for each time period. This was done ten different times, using a value

\textsuperscript{10}This represents the logic underlying delayed overshooting in Bacchetta and von Wincoop (2010).

\textsuperscript{11}The variance for the random term was computed using quarterly observations of ID data (between the USA and Germany, France, Japan, and the UK) from 1975 – 2007. First the variance for each ID time series was calculated. The variance for our constructed ID time series equals the mean of these four variances.
for $\lambda$ that ranged from 1.0 (perfect fx market efficiency) down to 0.10 (extreme inefficiency). The initial conditions were a zero value for ID, and $s^* = s = 1$. The first 2 values in the simulation were discarded. After the model was simulated in this manner, a Fama regression was estimated (for each assumed value for $\lambda$) using the calculated percentage change in the spot rate and the one period lagged value for ID.

The results are shown in Figure 1. When $\lambda = 1$ then the estimate of Fama’s $\beta$ is approximately +1, as in the model. As shown in (6a), if $\beta > 0$, then estimates of $\beta$ decrease as the fx market becomes less efficient. Figure 1 shows that initially $\beta$ decreases rapidly with $\lambda$, and becomes negative for values of $\lambda$ less than 0.8, as was concluded theoretically in section 2.3.2. Also, estimates of $\beta$ reach a minimum of approximately $-0.50$ when $\lambda$ is in the 0.3 to 0.4 range. Then smaller values for $\lambda$ tend to increase the estimated value for $\beta$, as is consistent with (6b).

In sum, the simulations show that the model can generate the UIP puzzle, and they suggest that the fx market does not have to be very inefficient ($\lambda < 0.80$) in order for estimates of Fama’s $\beta$ to be negative. The latter is consistent with Sarnos, et al. (2006) conclusion that any unexploited profit is relatively small.

3.2 Unstable Estimates of Beta and “Extreme Support”
In this stage of the simulations, the value for \( \lambda \) is assumed to depend positively on the absolute value for ID, as in equation (2). We assume discontinuous step changes in \( \lambda \) as follows: \( \lambda = 0.90 \) for the largest 10% of absolute values in ID; \( \lambda = 0.80 \) for absolute values of ID that fall between the largest 10% and largest 20%; \( \lambda = 0.70 \) for absolute values of ID that lie between the largest 20% and 30%, etc., all the way down to \( \lambda = 0.10 \) for the smallest (in absolute value) 10% of the IDs.

The results are shown in Figures 2 and 3. The former shows estimates of Fama’s \( \beta \) for rolling regressions, each of which uses 30 time periods of data. Estimates of \( \beta \) are extremely unstable, going from a minimum value of approximately -0.70 to a maximum of +0.10.\(^{12}\) As pointed out in section 2.3.2 instability arises because the relative frequency of large versus smaller absolute values for ID varies from one time interval to another. Hence, the degree of fx market inefficiency also varies over time, thereby yielding different estimates of Fama’s \( \beta \).

Figure 3 illustrates the results of simulations that focus on the non-linear econometric UIP models, i.e., the “extreme support” literature. As pointed out

\(^{12}\) These are the results from only one set of rolling regressions. The qualitative nature of the results remained unchanged when several other sets of rolling regressions were run.
above, this literature finds that outlier values for ID are typically associated with positive estimates for Fama’s $\beta$, while smaller (non-outlier) values for ID generate negative values for $\beta$. Figure 3 shows that our simulations are consistent with these results. The upper panel of Figure 3 shows estimates of $\beta$ from 100 simulations when Fama regressions were estimated using only the largest 20% of the absolute values for ID; all estimates are positive and range from approximately +0.10 to +0.55. The lower panel of Figure 3 shows estimated values for $\beta$ from 100 simulations wherein only the smallest 80% of the IDs were used; all but one are negative.

3.3 Delayed Overshooting

Section 2.3.3 showed that the model can be consistent with delayed overshooting with any given fixed degree of fx inefficiency, i.e., with $\lambda$ constant. Therefore, the simulations in this section assume that $\lambda$ is constant, but to test for the robustness of our results we use values of 0.9, 0.5, and 0.1. The procedure is as follows. After utilizing the initial conditions described above, the simulations assume that ID jumps up by +3 percentage points in period 3. In the first set of simulations ID decays at a rate of 20% per period, as in all simulations above. In order to focus on the question at hand, these simulations do not permit any other random fluctuations in ID after the initial shock. This ensures that ID decays in every period, thereby always yielding a “decaying ID effect” that works against
the “unexploited profit effect”. The results are shown by the solid lines in Figure 4. Initially, the spot rate declines (i.e., the dollar appreciates) sharply for all three values for \( \lambda \). In all cases, the maximum appreciation exists in period 6, which is 3 quarterly periods after the positive shock to ID. \(^{13}\) Then the dollar depreciates from the end of period 6 until somewhere between periods 13 and 18, depending on the value for \( \lambda \).

As a check on the robustness of our results, these simulations were repeated by using the same three values for \( \lambda \), but with the assumption of a faster rate of decay in ID, i.e. 40% per period. Recall that in each period there are two opposing forces at work on the spot rate. On the one hand, there is a tendency for the dollar to appreciate because of the existence of unexploited profit. On the other hand, there is a tendency for the dollar to depreciate because ID is consistently decaying. When the latter dominates the former, then the dollar stops appreciating and begins to depreciate, i.e., delayed overshooting occurs. Consequently, the faster rate of decay in ID in these simulations should generate: (a) a reversal at an earlier date, and (b) perhaps a smaller cumulative appreciation.

\(^{13}\) Eichenbaum and Evans (1995) find that the dollar persistently appreciates for more than two years, but Mark (2001, p. 196) finds that the maximum appreciation occurs between 8 and 10 months.
The dashed lines in Figure 4 show that (b) holds in all cases. Also, (a) holds precisely for $\lambda = 0.1$ and 0.5, where the persistent appreciation ends at the end of period 3. That is, the dollar appreciates only in period 3. i.e., only in the period when the positive shock to ID occurs. This is consistent with traditional thinking about UIP; that is, in this case, the UIP puzzle does not exist. When $\lambda = 0.9$ the dollar begins to depreciate in period 4, but then it appreciates slightly in period 5, before depreciating steadily after period 6. The important conclusion here is that the UIP puzzle and “delayed overshooting” do not exist if ID decays rapidly enough that the “unexploited profit effect” is always dominated by the “decaying ID effect”.

4. SUMMARY AND FUTURE WORK

Introducing fx market inefficiency in a general manner yields a UIP framework that nests the non-linear econometric UIP models as well as the “infrequent portfolio adjustment” model of Bacchetta and van Wincoop (2010). The inefficient fx market model appears to be consistent with many anomalies. A constant degree of inefficiency generates the UIP puzzle and the phenomenon of delayed overshooting. If, moreover, the market becomes more inefficient as the absolute value for the interest rate differential, ID, declines (which is consistent with the “limits to speculation” hypothesis) then the model generates unstable estimates of Fama’s $\beta$. Furthermore, Fama’s $\beta$ is positive for outlier values of ID,
but is negative for relatively smaller absolute values for ID. Finally, the model suggests a new missing variable in Fama regressions.

Obviously, the big question is: “How can the huge volume of fx transactions that occur not be large enough to move the spot rate sufficiently to satisfy UIP?” Bacchetta and van Wincoop (2010) present evidence that it is not profitable for investors to decide on the optimum amount of fx in their portfolios very often. This creates the possibility that the funds available for fx speculation in any one period might not be enough to satisfy ex ante UIP. Alternatively, Bacchetta and van Wincoop point out that any nonzero fx positions by banks are wiped out by the end of the working day. Thus, any net change in the spot rate from these activities might be trivial. A third possibility is that some investors are institutionally constrained with regard to the size of their speculative positions. Also, it is possible that liquidity constraints arise from macroeconomic considerations.

A final possibility deals with the fact that surveys of fx market dealers show that exchange rate expectations are strongly heterogeneous in nature. Froot and Frankel (1990) and Mark (2001) point out that heterogeneous expectations can greatly increase the volume of fx transactions. What has not been considered in the literature is that heterogeneous expectations might create a situation where there is a large quantity of both fx demanded, and fx supplied, but a relatively
small excess demand or supply. In such a case, the change in the spot rate is apt to be small.

In sum, this paper shows in a general way that many fx puzzles can be explained if the fx market is, at times, inefficient, and if the degree of inefficiency is positively correlated with the expected profit from carry-trade, as reflected by the absolute value for the home minus foreign nominal interest rate differential. Future work should thoroughly explore all possible reasons why the spot rate might not always move enough to satisfy ex ante UIP.
References


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Figure 1: Fama's Beta Against Lambda
Figure 3: Extreme Support
APPENDIX: DERIVATION OF (4c)

A1. $ds(t+k) = -\lambda E[\Pi(t+k)] = -\lambda \left\{ \Omega(t+k) - [s^* - s(t+k-1)] \right\}$.

a. $\Omega(t+k) = d\Omega(t+k) + \Omega(t+k-1)$

b. $s^* - s(t+k-1) = s^* - [s(t+k-2) + ds(t+k-1)] = [s^* - s(t+k-2)] - ds(t+k-1)$

A2. Insert A1a and A1b into A1 and use the fact that $d\Omega(t+k) = -\phi(t+k) \Omega(t+k-1)$ and

that $ds(t+k-1) = -\lambda \left\{ \Omega(t+k-1) - [s^* - s(t+k-2)] \right\}$ to get:

A3. $ds(t+k) = -\lambda \left\{ \Omega(t+k-1) - [s^* - s(t+k-2)] \right\} + \lambda \phi(t+k)\Omega(t+k-1) - \lambda \left\{ -\lambda \Omega(t+k-1) \right\} + \lambda \left[ s^* - s(t+k-2) \right]$.

Rearranging terms in A3 yields (4c).