Exam 1, Econ 672, Spring 2014

Total points are 100. But it counts 25% toward your final grade. So the effective total points are 25

Note: (i) discuss if you think the problem is unclear (eg, lacks some conditions); (ii) show me your work in details in order to get partial credits

Q1 (25 Points) Suppose there is an “almost” white noise process $e_t$ with the properties that
(i) $Ee_t = 0, \forall t$, (ii) $\text{var}(e_t) = \sigma^2_t, \forall t$, and (iii)

$$\text{cov}(e_t, e_{t-1}) = \rho, \forall t$$

$$\text{cov}(e_t, e_{t-j}) = 0, \forall j > 1, \forall t.$$ 

Now consider a first order moving average MA(1) process that uses the “almost” white noise $e_t$ as building block:

$$y_t = e_t + \theta e_{t-1}$$

(a): Find $E(y_t)$ (5 points)
(b): Find $\text{var}(y_t)$ (5 points)
(c): Find $\text{cov}(y_t, y_{t-1})$ (5 points)
(d): Is $y_t$ stationary? Why? (5 points)
(e): Is $y_t$ ergodic? Why? (5 points)

Q2 (25 Points) Jing uses the following R code to generate a time series

```r
n = 100
phi1 = 0
phi2 = 1
phi0 = 0.4
y = rep(0, n)
y[1:2] = 2
for (i in 3:n) y[i] = phi1*y[i-1] + phi2*y[i-2] + phi0 + rnorm(1)
plot(y, type="l", main="second-order equation")
```
(a): Explicitly write down the difference equation for \( y \) (5 points)
(b): Find the characteristic roots (10 points)
(c): Is \( y_t \) trending or not? Why (5 points)
(d): Does \( y \) have a fixed point as steady state? Why? (5 points)

Q3 (25 Points) Consider a simplified Samuelson multiplier-accelerator model:

\[
\begin{align*}
y_t &= c_t + i_t \\
c_t &= my_{t-1} \\
i_t &= ac_{t-1}
\end{align*}
\]

where \( y \) is GDP, \( c \) is aggregate consumption, \( i \) is aggregate investment, \( m \) is the marginal propensity to consume, and \( a \) is the accelerator.

(a): Explicitly write down the difference equation for \( y \) (5 points)
(b): Find the condition under which the steady state exists, and find the steady state (5 points)
(c): Find the condition under which the system converges to the steady state (5 points)
(d): Find the condition under which \( y \) will show sinusoidal (cyclical) pattern (5 points)
(e): Briefly discuss how to modify the model so that \( y \) becomes stochastic. You need to justify or provide intuition for your modification. (5 points)

Q4 (25 Points) Consider a third order autoregressive process AR(3):

\[
y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + e_t
\]

where \( e_t \) is white noise

(a): Explicitly write down the Yule-Walker equation for \( \gamma_j \equiv \text{cov}(y_t, y_{t-j}) \) (5 points)
(b): Please describe in detail how to simulate the impulse response function, and find the first three impulse responses starting with \( j = 0 \). (15 points)
(c): Find the condition under which the impulse response decays to zero as the lag increases (5 points)