Sketchy Key and Discussion to Old Exam 1

You are expected to provide more detailed answers in real exams

Q1a \( E(y_t) = E(\epsilon_t + \theta \epsilon_{t-1}) = 0 \)

Q1b \( \text{var}(y_t) = \text{var}(\epsilon_t + \theta \epsilon_{t-1}) = \sigma^2 + \theta^2 \sigma^2 \epsilon + 2 \theta \rho \). Notice that \( \text{cov}(\epsilon_t, \epsilon_{t-1}) = \rho \neq 0 \).

Q1c \( \text{cov}(y_t, y_{t-1}) = \text{cov}(\epsilon_t + \theta \epsilon_{t-1}, \epsilon_{t-1} + \theta \epsilon_{t-2}) = \rho + \theta \sigma^2 + \theta^2 \rho \). Notice that \( \text{cov}(\epsilon_{t-1}, \epsilon_{t-2}) = \frac{\rho}{\rho} \) since \( \epsilon_t \) is stationary.

Q1d yes, \( y_t \) is stationary because its mean and variance are constant, and autocovariance only depends on the lag.

Q1e yes, since \( \text{cov}(y_t, y_{t-j}) = 0, \forall j > 2 \). In words, \( y_t \) is erodic because the autocovariance decays toward zero sufficiently fast.

Q2a \( y_t = y_{t-2} + 0.4 + \epsilon_t \), where \( \epsilon_t \) is white noise

Q2b The characteristic equation is \( x^2 - 1 = 0 \), so characteristic roots are \( x_1 = 1, x_2 = -1 \). There is only one unit root.

Q2c Yes, \( y_t \) is trending because \( \phi_1 + \phi_2 = 1, \phi_0 \neq 0 \). See HW1.

Q2d No. Actually we should try \( \bar{y} = ct \), where \( c \) is unknown. That is, in this case the steady state is a linear trend, not a fixed point, because one unit root is present. (We should try quadratic trend as the steady state if two unit roots are present)

Q3a \( y_t = my_{t-1} + amy_{t-2} \)

Q3b Under the condition \( 1 - m - am \neq 0, \bar{y} = 0 \).

Q3c Both characteristic roots should be less than one in absolute value

Q3d \( m^2 + 4am < 0 \), under which there are two conjugate complex characteristic roots, which give rise to sinusoidal behavior.

Q3e There is no unique answer. For example, we may add a white noise random shock \( \epsilon_t \) to the consumption function, which may represent something like animal spirit that affects consumption in a random fashion.
Q4a Since this is a five-point question, you can just say $\gamma_j = \phi_1 \gamma_{j-1} + \phi_2 \gamma_{j-2} + \phi_3 \gamma_{j-3}, \forall j \geq 3$. Notice that this equation does not hold for $j = 0, 1, 2$.

Q4b Let all shocks be zero except $e_t = 1$. Then we can simulate $y_t = 1; y_{t+1} = \phi_1; y_{t+2} = \phi_1^2 + \phi_2$

Q4c We can show the impulse response $\frac{dy_{t+1}}{de_t}$ follows the same deterministic difference equation as $y$. That means, in order for the impulse response to decay to zero—the steady state, we require that all the characteristic roots be less than one in absolute value.