Did the Soviets collude? A statistical analysis of championship chess 1940–1978

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ABSTRACT

We expand the set of outcomes considered by the tournament literature to include draws and use games from post-war chess tournaments to see whether strategic behavior can be important in such scenarios. In particular, we examine whether players from the former Soviet Union acted as a cartel in international all-play-all tournaments – intentionally drawing against one another in order to focus effort on non-Soviet opponents – to maximize the chance of some Soviet winning. Using data from international qualifying tournaments as well as USSR national tournaments, we consider several tests for collusion. Our results are inconsistent with Soviet competition but consistent with Soviet draw-collusion that yielded substantial benefits to the cartel. Simulations of the period's five premier international competitions (the FIDE Candidates tournaments) suggest that the observed Soviet sweep was a 60%-probability event under collusion but only a 25%-probability event had the Soviet players not colluded.

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Research into how tournament structure affects effort in athletic competition has generated many insights into the problems of optimal compensation in labor and management. In much of this literature, outcomes are monotonic functions of effort plus an error term (cf. Ehrenberg and Bognanno, 1990), match-ups end in either wins or losses, and strategic behavior by players is accordingly considered unimportant. Many important economic scenarios, though, are better characterized by the presence of a third outcome that is analogous to a draw, and the existence of such a draw outcome can introduce the potential for strategic behavior. In this paper, we test whether Soviet chess players exploited the existence of the draw and round robin (i.e., all-play-all) formats, effectively acting as a cartel in international tournaments following World War II. Our findings suggest that the existing tournament literature may have fewer real world applications than has been appreciated and that important caveats may apply.

"Draws" in life are commonplace, and the presence of a draw strategy complicates the problem of encouraging effort. For instance, a player may have the choice of more and less conservative strategies with different chances for a draw. Risk-averse players may pursue less aggressive strategies, trading expected payout for reduced variance.[1] Furthermore, it is not

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[1] Consider the following hypothetical. Assume that a large automotive company wishes to motivate one of its divisions to innovate by offering a very high reward in the event of a new design success, but consistent with much of the tournament literature, the company also proposes no reward or even punishment if the proposed design fails. Under certain assumptions, the division would do its best to produce a terrific design but only if outcome is monotonic in effort. On the other hand, suppose the division had two options: a low-risk design (the draw option) with little chance of being a "home run" but also little chance of being a failure or a high-risk design (the play for the win strategy) with a higher probability of leading to a big success but also a

unreasonable to think that many real-world tournament settings are more akin to the repeated play against many opponents that one finds in the round robin format than to the elimination rules of a knockout format. The combination of the draw option with something similar to an all-play-all set of rules creates the possibility that collusion among a subset of players may upend at least some of the predictions from tournament theory.

In our setting we begin by noting that tournament chess is not simply stressful but notoriously tiring. We hypothesize that very strong players who wish to collude in round robin formats can improve their performance against other players by agreeing to early or prearranged draws. Assuming for simplicity that the expected outcome of a game between players of equal strength is half a point each, a peaceful pair of agreed draws produces the same outcome with less effort and risk than taking a win and loss apiece.

The literature on the economics of tournaments within sporting events is now well established. For our purposes the most relevant work has focused on the importance of tournament structure in motivating effort and in providing incentives for good play (see the overview by Szymanski, 2003). In most cases the applications have been to athletic competitions, whether individual (golf or tennis) or team sports (basketball or soccer). There is also a related literature on the problem of collusion and of cheating in sports that overlaps with this work.[2]

Chess has numerous strengths for the purposes of econometric analysis. First, the outcomes are clear and objective: a win, a draw, or a loss. Moreover, a perfect record of all games is available for virtually all important championship and high-level tournament games of the modern era. Most important of all is that there exists a rating system that is a precise and accurate reflection of the performances of players and an excellent indicator of the relative strengths of players. These ratings are the best unbiased estimates of relative strengths, and the differences in ratings correspond to the likelihood that the stronger player will defeat the weaker (cf. Elo, 1978). These Elo-style ratings have since been applied not only to other sports but also to studies of revealed preference rankings in college selection (cf. Avery et al., 2005).

The question of Soviet collusion during the period when the USSR dominated the struggle for the world championship has been one of the long-standing debates in the chess world. The best-known (but not unique) criticism was aired by Bobby Fischer some weeks after the 1962 Candidates Tournament in Curacao in the magazine *Sports Illustrated* in the article "The Russians Have Fixed World Chess" (cf. Timman, 2005).[3] In it, he claimed that three of the Soviet players (Paul Keres, Tigran Petrosian, Efim Geller) had agreed to pre-arranged draws among themselves in order to conserve energy and to permit themselves to play to the fullest against the non-Soviet competitors. A fourth player (Viktor Korchnoi) was supposedly forced to throw games to the others to minimize the chance of a non-Soviet (probably Fischer) winning the tournament.[4] These allegations were immediately denied by the Soviets, and many observers have noted that Fischer performed so poorly in that tournament that any collusion was unlikely to have been the cause of his defeat (Timman).[5] Nonetheless, the World Chess Federation (Fédération Internationale des Échecs or FIDE), the governing body of the world chess organization, was sufficiently concerned to modify the system for selecting the challenger for the world championship from 1964 onwards, changing the penultimate Candidates tournament from round robin to a knockout format.[6] We exploit this change in tournament format to quantify the extent of Soviet outperformance of rating that was due to draw-collusion as opposed to other potential Soviet advantages.

Prior analyses of the question of Soviet collusion have turned on the evaluation of individual games and the judgment as to whether some were deliberately "lost" or "drawn" in favorable positions.[7] For the most part there has been no systematic analysis of the pattern of wins and draws in the tournaments themselves, though all have noted that there was indeed a large number of draws between the three leading Soviet players in Curacao 1962 (Timman, 2005).[8]

The problem of detecting collusion has naturally received much attention in industrial organization, and our identification strategy has a strong parallel there. Our approach when applied to games in round robin events is most in line with the literature that makes comparisons to a broadly acceptable conduct benchmark. For example, Bresnahan (1987) uses the approach to great effect, coupling the benchmark that a firm will set prices "collusively" for all products in its portfolio

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with the observation that firms may cluster products in some areas of product-space and achieve a local near-monopoly. He then implicitly compares inferred markups of autos that share a portfolio and similar product characteristics with those of autos of similar characteristics but differing ownership. Our identification comes from the observation that a Soviet cartel has no bearing in an all-Soviet tournament. The fact that the leading Soviet players regularly competed with each other in Soviet national tournaments implies that those tournaments can serve as our competitive benchmark as we consider international play. This approach is sufficient for the identification of our predictions regarding the likelihood of draws, but in much of our analysis we supplement it with data on player quality, namely ratings that make use of the entire history of play. The absence of such an objective measure of quality is in part what typically complicates the detection of collusion in industrial organization, and the powerfully predictive ratings in chess suggest that the question of collusion may be convincingly answered here.[9] In so doing we hope to highlight the potential importance of draws and draw-like scenarios in the tournament literature and the real world.

1. The structure and history of the World Championship 1948–1962

Prior to the Second World War, the World Chess Championship was considered the private property of the world champion. He alone could determine which players could challenge him. Championship matches required the solicitation of private funds for the challenger and the champion, and the terms of the match were subject to negotiation. If the champion were dethroned, the new champion would have the right to impose conditions on future challengers. Attempts were made to regulate the championships, but for the most part these rules were unenforceable because of the private nature of the title.

The death of the world champion Alexander Alekhine in 1946 allowed the professional organization FIDE to seize control of the world championship and regulate future competitions. They instituted formal regulations for awarding official titles of international master (IM) and international grandmaster (IGM or more commonly GM) and imposed a structure on acquisition of the world title. To replace Alekhine, a special quintuple round robin tournament split between The Hague and Moscow was held with five invited players in 1948. The players were Mikhail Botvinnik, Max Euwe, Keres, Samuel Reshevsky, and Vasily Smyslov, and Botvinnik emerged as world champion. From that point on, the championships were put on a triennial cycle. The point of the cycle (based upon local competitions arranged by geographic region) was to select players who would play in a qualifying tournament called the Interzonal. The top six players would then join participants other than the champion from the last qualifying cycle (the specific details of qualification and the exceptions therefrom are not important) in a Candidates tournament, which was a multiple (two or four) round robin event. The winner of this tournament was designated the official Challenger who would then play a 24-game match for the world championship against the incumbent. Should the incumbent be defeated, he was entitled to a rematch. The Candidates tournaments were considered to be difficult and grueling affairs in which the physical stamina of the players could have affected the quality of their play, raising the concern about prearranged draws.

Prearranging draws did not violate any explicit rules because there was no way to enforce a rule that restricted such behavior. Despite its equal lack of enforceability, deliberately losing a game was more likely to be considered outright fraud. Nonetheless all forms of prearrangement were viewed as unethical or worse. A group of leading players could conceivably increase their chances of success by playing conservatively among themselves at first and then, if necessary, perhaps even throwing games to whichever member of the group happened to be "in form" as the tournament progressed. In practice, only the USSR had a consistently large enough group of players in the FIDE qualifying (Interzonal and Candidates) tournaments for such a plan to be feasible.

2. The model

We consider two hypotheses that describe player behavior in chess tournaments. Our null is that individuals choose strategies and exert effort to maximize their own likelihood of winning a particular tournament. We will abstract away from some real-world circumstances and restate this as individuals acting to maximize the probability of winning the game that they are playing.[10] Our alternative hypothesis is that Soviet players act as a cartel in FIDE events, acting within constraints to maximize the probability that some member of the cartel wins the tournament.[11] Soviets in non-round robin FIDE events and non-Soviets in all events, however, act to maximize the probability of winning the game that they are playing.

Within the context of the draw collusion explained above, there are three testable implications of the collusion hypothesis.

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[9] Indeed, none of the previous papers on collusion in sports have quality/performance ratings that can rival the modified Elo-Sonas ratings used in this paper. For example, Duggan and Levitt (2002) rely on small-sample win–loss records that take little account of the strengths of all the participants and do not allow fine distinctions both across players and intertemporally.

[10] This simplifying assumption runs counter to the observation that a player with a commanding lead going into the last round of a tournament is much likelier to play for a draw.

[11] One plausible such constraint is that, within the cartel, each player wants to win the tournament. Consequently no player is willing to throw matches to other players in the cartel.

1. Games that involve two Soviets, are in FIDE round robin events, and end in draws should end with fewer moves than other games that end in draws.
2. Games that involve two Soviets and are in FIDE round robin tournaments should be likelier to end in draws than other games (i.e., games in all-Soviet tournaments (URS, following International Olympic Committee country codes) and games in FIDE knockout tournaments).
3. Soviet players should outperform their ratings in FIDE round robin tournaments when playing a non-Soviet opponent, and this outperformance should exceed that of Soviets playing non-Soviets in FIDE knockout tournaments.

We address the first prediction using a simple comparison of the number of moves using the sample of games that end in draws. The second prediction is approached in two ways. First, we compare the likelihood of draws in games involving the same Soviets playing particular colors at roughly the same time across FIDE and URS games. Second, we apply an ordered probit to Soviet and non-Soviet players to consider whether the draw thresholds for Soviet-on-Soviet play in FIDE round robin events systematically differ from those of other events. We test the third prediction by returning to the ordered probit estimates. To confirm that such outperformance stems from draw collusion, we compare the estimated Soviet outperformance to any outperformance that arose in later knockout tournaments where draw-collusion was impossible. We also explore the nature of the benefits from draw-collusion by examining how variables capturing player fatigue affect outcomes over the course of tournaments.

White's relative advantage over Black will be dependent upon the comparative mastery of the two players. We capture this comparison with the variable *RatingDif*, White's rating less Black's rating.[12] To explore our third prediction, we would ideally have data on the relative fatigue of the two players, or more plausibly the difference between players' cumulative moves at the start of a game. If draw collusion is important and fatigue-avoidance is a primary mechanism, we expect Soviet players to outperform most visibly toward the end of a tournament and likewise expect that such outperformance will be minimal at the start of a tournament. While we will eventually investigate such fatigue measures, we first consider the concept of *SovietDif* as a cruder measure of this relative advantage. Let *WhiteSov* be a binary variable equal to 1 if White is Soviet, and let *BlackSov* be defined analogously. *SovietDif* is then *WhiteSov*–*BlackSov*, taking the values of 1, 0 or −1. *KO* is a binary variable equal to 1 if the game is part of a tournament with a knockout format; it will be interacted with *SovietDif* to determine how much of Soviet outperformance was due to draw collusion.

The most obvious issue regarding the threshold against which we compare this relative advantage of White is the inherent advantage of White's position in moving first. We correspondingly do not expect these base thresholds to be symmetric and impose no such restriction. We further parameterize these thresholds as potentially depending upon whether a game is part of a FIDE tournament (*FIDE*) and whether both players are Soviets and the game is part of a FIDE tournament (*BothSov* × *FIDE*). The former parameter measures the extent to which all players are likelier to draw (play more conservatively) in FIDE events, and the latter parameter captures the collusion hypothesis's first prediction. The triple interaction *BothSov* × *FIDE* × *KO* will enable us to see how draw behavior in FIDE games with two Soviets differed across knockout and round robin games. Preliminary results indicated that a symmetry assumption on these margins was benign, and parameters reflect this symmetry restriction.

Formally, let RAW_{jkt} denote player j's relative advantage as White over player k (Black) at time t. This relative advantage is dependent upon both observed (*RatingDif*, *SovietDif*, *SovietDif* × *KO*) and unobserved characteristics: $RAW_{jkt} = X_{jkt}\beta + \varepsilon_{jkt}$, where $\varepsilon \sim N(0, \sigma^2)$. Thresholds depend only upon observed characteristics (*FIDE*, *BothSov* × *FIDE*, *FIDE* × *BothSov* × *KO*). Let the winning threshold be denoted $\gamma_{jkt}^{\text{Win}} = \gamma^{W} + Z_{jkt}\gamma$, so that player j wins as White if $RAW_{jkt} > \gamma_{jkt}^{\text{Win}}$. The losing threshold is comparable and denoted as $\gamma_{jkt}^{\text{Lose}} = \gamma^{L} - Z_{jkt}\gamma$, so that player j loses as White if $RAW_{jkt} > \gamma_{jkt}^{\text{Lose}}$. Player j then draws as White if $\gamma_{jkt}^{\text{Lose}} < RAW_{jkt} < \gamma_{jkt}^{\text{Win}}$.[13]

These parameters are only identified up to scale, and we accordingly normalize the variance of the normal disturbance to $\sigma = 1$. We estimate these parameters using maximum likelihood.

3. The data

Our data were supplied by Jeff Sonas of Chessmetrics.com and include all games of FIDE events and the Soviet (URS) national championship from 1940 to 1978, for a total of 7355 games. We then supplemented this data for games before 1965 with information regarding game ordering and ply-count from Chessbase's Mega database 2006. An observation of a game includes the tournament, the players' names, their ratings, which player was White, the ply number (half-moves) at game conclusion, and the outcome. In light of the potential for player movement or defection, whether a player is Soviet reflects the residence of a player at the time of a tournament. We have complete observations for all early round robin FIDE games and about 40% of early URS games. The most common missing variables are ply-count and the round of the game, and we therefore perform our ply-count regressions and fatigue-based ordered probit on smaller samples.

What makes this data set so valuable for our purposes is the continuity and frequency of the ratings derivation. Ratings in chess that make use of rigorous statistics to produce good estimates of relative player strength are now relatively common,

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[12] We discuss these ratings in greater detail in our review of the data.
[13] Ties of RAW and thresholds are assumed to occur with zero probability.

Table 1
Variables.

	Definition	Obs	Mean	S.D.
PlyCountDraws	Half-moves in games that end in draws	2091	73.33	34.67
RatingDif	(White rating − Black rating)/100	7355	0.00	0.89
SovietDif	WhiteSov − BlackSov	7355	0.00	0.44
FIDE	1 if FIDE event, 0 o/w	7355	0.43	
BothSov	1 if both players Soviet, 0 o/w	7355	0.66	
FIDE × BothSov	Interaction	7355	0.09	
KO	1 if game part of knockout format event	7355	0.09	

but comparing ratings across different time periods is often complicated by idiosyncratic changes (cf. Elo, 1978, for the pioneering discussion). Sonas uses a weighted and padded simultaneous performance rating formula throughout our sample and updates this rating monthly instead of annually, as is more common (cf. Sonas, 2005, 2006). This specification has been optimized for predictive power for games between world-class players. While the formula is given in Appendix B, there are three primary differences between these Sonas ratings and standard Elo-based ratings. Besides a linear framework that appears to dominate in predictive power the more common logistic specification, Sonas ratings weight more recent games more heavily than distant games (looking back 4 years) and are padded to reward players who play more games.

Retrospective grading allows Sonas to establish ratings that are unbiased estimates of the "true" relative strengths of players. This rating requires a minimum number of observed games to construct, and games without Sonas chess-ratings for both players are dropped from the sample. While this leads to the omission of a few Interzonal games, the vast majority of dropped games are from URS championships. These omissions will presumably increase the average observed skill of Soviet players in URS championships and, thereby, make our comparison to FIDE events even more compelling.

Table 1 includes variable names, definitions, and summary statistics of our primary set of observations. Variables that are incorporated into White's relative advantage are at the top. As one would expect from the random way in which White is assigned, both *RatingDif* and *SovietDif* have mean zero. From the threshold variables below, we can see that 43% of our full sample's games are from FIDE tournaments, and 9% of the total games involve two Soviets playing in a FIDE event. Roughly 8.5% of observed games occur in tournaments with a knockout format (late sample FIDE Candidates or World Championships). From games for which we have all information, ply-count variable indicates that the typical draw ended after 36 moves, as a move is composed of two plies (half-moves).

4. Results

"Hard-fought" games that end in draws are more likely to last longer than collusive or pre-arranged draws. The latter are more likely to be agreed to at an earlier stage when the position on the board is still not fully resolved and it is not clear than one player should win. At a later stage, the likelihood is much greater that the position will clearly favor one or the other player (to the point where it would be implausible for grandmasters to agree to a draw when one is in a strongly dominant position). While raw comparisons fail to account for the variation in player skill that may exist across the Soviet national and FIDE tournaments, they will be suggestive of later results.

Using the sample of games that ended in draws and for which we observe ply-counts ($n = 2091$), we begin our evidence with a comparison of draw lengths in Table 2A. Relevant *t*-statistics (based upon spherical standard errors) are provided for comparisons. At least two conclusions can be drawn. One, there is some evidence that draws took marginally more plies (3) in URS championships than in FIDE events. Two, there is strong evidence that draws involving two Soviets in FIDE events took significantly fewer (13) plies than FIDE event draws involving at least one non-Soviet. The cartel's draws in FIDE events are estimated to occur eight moves sooner than draws in URS championships. In major chess tournaments, playing a mere four or five additional moves (10-plies) at a critical stage of the middle game requires considerably more effort, as well as engendering greater risk. The results therefore indicate differences that are of substantive and not simply statistical significance.

Table 2A
Comparison of means (ply-counts of draws ($n = 2091$)).

	n	Mean
FIDE	900	69.96
FIDE|BothSov = 0	714	72.64
FIDE|BothSov = 1	186	59.68
URS	1191	75.87
	Dif	|t|
−FIDE|BS = 0 < URS	3.22	1.98
−FIDE|BS = 1 < FIDE|BS = 0	12.97	4.58
−FIDE|BS = 1 < URS	16.19	5.98

Table 2B

Comparison of means (draw rates ($n = 7335$)).

	n	Mean
FIDE	3188	0.528
FIDE|RR	2560	0.520
FIDE|KO	628	0.559
URS	4167	0.500
FIDE|RR|BothSov = 0	2203	0.498
FIDE|RR|BothSov = 1	357	0.658
FIDE|KO|BothSov = 0	322	0.556
FIDE|KO|BothSov = 1	306	0.562
	Dif	|t|
-FIDE|RR|BS = 0 < URS	0.003	0.2
-FIDE|RR|BS = 1 > URS	0.158	5.75
-FIDE|RR|BS = 1 > FIDE|RR|BS = 0	0.161	5.65
-FIDE|KO|BS = 1 > FIDE|KO|BS = 0	0.006	0.156

Table 3

Almost ideal draw rate comparison. Comparison of draw rates in FIDE games vs. matching URS games; same players, same colors, varying size of window of URS games; bigger windows increase sample and precision, lessen appropriateness of comparison.

Window, N	URS DR	FIDE+	Window, N	URS DR	FIDE+
$w = \pm1$, $n = 67$	0.612 (0.057)	0.052 (0.081)	$w = \pm6$, $n = 226$	0.604 (0.028)	0.069 (0.039)
$w = \pm2$, $n = 136$	0.608 (0.040)	0.028 (0.056)	$w = \pm7$, $n = 228$	0.599 (0.027)	0.076 (0.038)
$w = \pm3$, $n = 168$	0.609 (0.035)	0.040 (0.049)	$w = \pm8$, $n = 229$	0.593 (0.027)	0.080 (0.038)
$w = \pm4$, $n = 205$	0.594 (0.031)	0.064 (0.044)	$w = \pm9$, $n = 230$	0.591 (0.026)	0.079 (0.037)
$w = \pm5$, $n = 220$	0.601 (0.029)	0.067 (0.041)	$w = \pm10$, $n = 232$	0.587 (0.026)	0.083 (0.037)

Estimated standard errors in parentheses.

A similar story appears upon examining draw rates, shown in Table 2B. Aggregate draw rates across the FIDE and URS tournaments and across FIDE tournaments of differing format are similar, but this may camouflage differences that occur depending upon whether both players are Soviet. We therefore consider the draw rates in both formats of FIDE events conditioned on whether both players in a game were Soviet. This result is much more striking: 66% of round robin FIDE games involving two Soviets ended in draws, compared to a 50% draw rate for round robin FIDE games in which at least one player was not Soviet. This contrasts with comparable figures for knockout FIDE games (56.2% vs. 55.6%). This, of course, matches the second prediction of the Soviet collusion hypothesis.

The above comparison, though highly significant, is not quite perfect. Draw rates in URS tournaments may have been depressed as dominant Soviet players took advantage of weaker Soviet opponents and played more aggressive strategies. To address this concern, we perform a comparison that is almost ideal. That is, we compare each early round robin FIDE game (1948–1964) between two Soviets that ended in a draw to the draw-likelihood of games at around the same time involving the very same two players playing the same colors in all-Soviet tournaments. Because not all Soviets play one another in any given URS tournament, the sample size grows with the window around the FIDE game. Larger windows should lead to more precise estimates but weaken the quality of the comparison as their relative strengths might have changed significantly over the larger time period.

Table 3 displays the results of this comparison for varying window sizes. As speculated, the URS draw rate in this setting is substantially higher than the overall URS draw rate (60% vs. 46%). Even against this higher draw rate, however, the difference from the FIDE draw rate is always positive. This difference stabilizes around the value of 0.07 when windows are at least ±4 years and becomes statistically significant at conventional levels when windows exceed ±6 years. Given that the largest sample size is 232, this particular measure seems to offer especially compelling evidence in favor of the hypothesis of draw collusion.

Estimates from the ordered probit using the full dataset can be found in Table 4(I–II).[14] All parameters are precisely estimated. In addition to the obvious results of the difference in ratings being critical, results indicate the presence of a first-mover advantage ($\gamma^W < -\gamma^L$) and marginally more conservative play in FIDE events than in URS events. The inclusion of the four Soviet cartel variables significantly decreases the log-likelihood function ($p = 0.00003$). Furthermore, both predictions of the Soviet collusion hypothesis are borne out. Soviets did outperform their ratings in FIDE events, and Soviets were likelier to draw against each other in FIDE events than in URS events. The magnitude of these estimates suggests that the

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[14] We also estimated this ordered probit allowing for a knockout interaction among the relative advantage of White variables and a knockout indicator for the threshold. Coefficients on these variables were not significantly different from zero, and other estimates did not change when the restriction was imposed. In separate regressions, we found no evidence of a change in Soviet conduct after Curacao (when Fischer's very public allegations broke), and so we assume constant Soviet conduct throughout.

Table 4
Ordered Probit.

	I	II	III	IV	V	VI
RatingDif	0.42 (0.02)	0.40 (0.02)	0.45 (0.03)	0.45 (0.03)	0.44 (0.03)	0.45 (0.03)
SovietDif	–	0.17 (0.04)	0.21 (0.05)	0.20 (0.05)	0.19 (0.05)	0.19 (0.05)
SovietDif × KO	–	−0.17 (0.08)	–	–	–	–
FatigueDif						
Games	–	–	–	(0.082 (0.039)	–	–
Ply/1000	–	–	–	–	(0.346 (0.151)	–
BothSov	–	–	–	–	–	0.008 (0.009)
Thresholds						
Win	0.53 (0.02)	0.53 (0.02)	0.59 (0.04)	0.59 (0.04)	0.59 (0.04)	0.58 (0.04)
Lose	(0.92 (0.02)	(0.92 (0.02)	(0.93 (0.04)	(0.93 (0.04)	(0.93 (0.04)	(0.93 (0.04)
FIDE?	0.06 (0.02)	0.04 (0.02)	–	–	–	–
FIDE × BothSov	–	0.22 (0.05)	0.24 (0.06)	0.23 (0.06)	0.24 (0.06)	0.24 (0.06)
FIDE × BothSov × KO	–	(0.18 (0.07)	–	–	–	–
Log-lik	7107.69	7086.52	1604.32	1601.93	1601.81	1603.95
n	7355	7355	1751	1751	1751	1751

Standard errors in parentheses.

average impact of the effort avoided by the Soviets in FIDE events was equivalent to 42.8 points in the ratings (a.s.e. 9.7 points).

This estimate is strikingly large, and one might reasonably wonder if it is aggregating factors besides the benefit of draw collusion. For example, if Soviet players in international play (but not URS play) receive treatment and resources (such as state-sponsored pre-tournament preparation and training) that are superior to that of other countries' players, attributing the entire outperformance estimate to draw-collusion is an obvious overstatement.[15] The knockout interactions from the ordered probit address exactly this concern. Any sources of Soviet outperformance that did not stem from draw collusion will presumably persist after the format change, so any differences across formats can be attributed entirely to draw collusion. Both knockout interactions are statistically less than zero. For the thresholds, this reflects the earlier result that there was a much smaller difference in draw rates between games with two Soviets and the other games under the knockout format than under the round robin format. The fact that the absolute value of the coefficient on *SovietDif × KO* is approximately the same as that on *SovietDif* supports the view that Soviet outperformance in international round robin formats stemmed primarily, and perhaps entirely, from the benefits of draw-collusion.

We continue our estimation by considering the potential mechanisms through which Soviets outperformed against non-Soviets in FIDE events. Suppose that players began a tournament with a plan for effort expended for each game and draw-collusion helped Soviets by enabling them to limit their exertion against fellow Soviets and to concentrate more fully against non-Soviets. Under such a mechanism, we expect to see no systematic difference in the extent of Soviet outperformance at different rounds of a FIDE tournament. If, however, draw-collusion allowed Soviet performance through the simpler mechanism of players being less fatigued as they advance, then we expect Soviet outperformance to be less early in a tournament and greater at a tournament's end.[16]

As the necessary round data were scarce for URS events and later FIDE events, we re-estimate the ordered probit model using only round robin FIDE games prior to 1965 (n = 1751) as our new baseline. This necessitates dropping the knockout interactions. In addition to dropping the FIDE indicator from the threshold variables, we also include a variety of proxies for the difference of players' fatigue levels. Table 4 (III–VI) displays results. Estimates shown in (III) replicate our earlier results omitting any *FatigueDif* variables. Columns (IV–VI) then separately consider the fatigue proxies of prior games played in the tournament (to capture the advantage of bye-rounds), prior plies (in 1000s), and prior games against fellow cartel members.[17] Similar to *RatingDif* and *SovietDif*, these variables are differenced between players before being incorporated as regressors into the relative advantage of White. We expect the first two variables to have negative coefficients and the third to have a positive coefficient.

While all estimated coefficients have the expected sign, none greatly reduce the log-likelihood. Only the first two variables are significant at the 95% confidence level, and none are significant at the 99% confidence level. Having played one fewer game than one's opponent is estimated to yield an advantage equivalent to an additional 18 rating points. The estimate on ply-count suggests that having a 100 lower cumulative ply-count (50 moves) offered an advantage of about 8 rating points. Given the arguable importance of these magnitudes, we tentatively conclude that fatigue reduction played at most a complementary role in causing Soviet outperformance. The primary purpose of the Soviet cartel was to allow Soviet players to concentrate their effort more fully upon non-Soviet opponents regardless of a game's temporal location in a tournament.

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[15] The same reasoning would hold for psychological incentives (e.g., fear of punishment for losing in international settings).

[16] We tested for differential Soviet fatigue rates (as might be suggested by their intensive training regimens) but could find no evidence supporting such differences being important.

[17] Byes play no role in the Soviet collusion story and are included merely to show whether fatigue generally matters in these tournaments.

Table 5
Implications of estimates.

A. Two evenly matched non-Soviets in FIDE event	
Pr(White wins)	0.2852
Pr(White draws)	0.5464
Pr(White loses)	0.1684
B. Two evenly matched Soviets in FIDE event	
Pr (White wins)	0.2168
Pr (White draws)	0.6634
Pr (White loses)	0.1198
C. Evenly matched Soviet and non-Soviet (Soviet playing White)	
Pr (White wins)	0.3455
Pr (White draws)	0.5254
Pr (White loses)	0.1292

Table 6
Monte Carlo of Budapest 1950 Candidates. 10,000 simulations using baseline ordered probit estimates (Table 4-II). 10-Player, double round robin (play each opponent 1× as White and 1× as Black).

Players	Rating	Sov?	ActScore	Cartel		No Cartel	
				E (PredScore)	% win	E (PredScore)	% win
Boleslavsky	2722.17	Y	12	9.30	0.092	9.03	0.071
Bronstein	2753.67	Y	12	9.98	0.214	9.77	0.181
Flohr	2659.34	Y	7	8.01	0.011	7.55	0.008
Keres	2721.96	Y	9.5	9.30	0.088	9.00	0.071
Kotov	2752.53	Y	8.5	9.96	0.212	9.73	0.177
Lilienthal	2660.19	Y	7	8.02	0.013	7.57	0.009
Najdorf	2741.06	N	9	8.76	0.060	9.47	0.124
Smyslov	2760.36	Y	10	10.09	0.249	9.90	0.210
Stahlberg	2731.50	N	8	8.53	0.042	9.23	0.098
Szabo	2709.60	N	7	8.06	0.020	8.75	0.052
				Soviet win?	0.879		0.727

Notes: 2276 ties are predicted under collusion (78% 2-way, 18% 3-way, 4% 4-way or more), 2139 ties are predicted under no-cartel (80% 2-way, 17% 3-way, 3% 4-way or more).

Table 5 lists some implications of the full-sample, no-fatigue parameter estimates on the probabilities of game outcomes. First consider the baseline of two evenly matched non-Soviets meeting in a FIDE event. The model's estimates indicate that White has a 28% chance of winning, a 55% chance of drawing, and a 17% chance of losing. This compares to the model's implications of expected draw rates in games between two Soviets that mimic the raw draw rates: 48% in URS events and 66% in FIDE events. Assuming Soviet collusion, the probability of an evenly matched Soviet playing White winning against a non-Soviet in a FIDE event rises from 28% to 35%.

5. Simulations

The evidence thus far does not speak to the question of whether this Soviet collusion had important impacts on tournament outcomes. For example, it could be that the Soviets were so dominant that this collusion only marginally increased the likelihood of some Soviet winning a FIDE tournament. To consider this question, we use the original estimates to simulate the five Candidates tournaments that occurred from 1950 to 1962 and determined the challenger to the world champion.

Observed characteristics and the model's estimates imply the probabilities of the three outcomes. Using a draw from the uniform distribution to simulate the outcome of each game, we then re-play each Candidates tournament 10,000 times.[18] The tournament's winner is the player with the most points (1 pt for win, 1/2 pt for draw, 0 pt for loss).[19] We conclude by considering a competitive counterfactual in which Soviets do not systematically outperform their ratings and they are no more likely to draw against Soviets than against non-Soviets.

Table 6 shows actual and simulated outcomes as well as player characteristics for the double round robin 1950 Candidates in Belgrade. The limited range of the ratings gives additional perspective regarding the size of the estimated Soviet collusive advantage (43 points). Soviet dominance is also apparent, as seven of the 10 participating players are from the USSR. Under the collusive hypothesis, Smyslov was the favorite (winning 25% of the time), but the eventual winner Bronstein and Alexander Kotov were not far behind (each winning 21% of the time). From the Soviet cartel's perspective, a Soviet would have won

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[18] For example, an observed game's thresholds and relative advantage for White might imply a 10% chance of White losing, a 50% chance of White drawing, and a 40% chance of White winning. Letting *e* be the uniform draw, the simulated game ends in a loss if *e* < 0.1, a win if *e* > 0.6, and a draw otherwise.

[19] Two-way ties are resolved with the best of twelve format used in the 1950 Candidates between Isaac Boleslavsky and Bronstein, and ties involving more than two players are resolved using a double round robin format. Ties at the end of these tie-breakers are resolved using a random number generator.

Table 7
Monte Carlo of Zurich 1953 Candidates. 10,000 simulations using baseline ordered probit estimates (Table 4-II). 15-Player, double round robin (play each opponent 1× as White and 1× as Black).

Players	Rating	Sov?	ActScore	Cartel		No Cartel	
				E (PredScore)	% win	E (PredScore)	% win
Averbakh	2645.01	Y	13.5	12.14	0.002	11.29	0.001
Boleslavsky	2722.33	Y	13.5	14.57	0.069	14.00	0.042
Bronstein	2723.87	Y	16	14.65	0.073	14.06	0.043
Euwe	2687.61	N	11.5	11.90	0.002	12.79	0.008
Geller	2734.03	Y	14.5	14.95	0.103	14.41	0.062
Gligoric	2707.39	N	12.5	12.55	0.006	13.44	0.019
Keres	2721.02	Y	16	14.57	0.068	14.00	0.047
Kotov	2727.00	Y	14	14.72	0.080	14.15	0.049
Najdorf	2753.04	N	14.5	14.20	0.059	15.09	0.130
Petrosian	2717.44	Y	15	14.40	0.055	13.81	0.034
Reshevsky	2780.99	N	16	15.13	0.148	16.03	0.286
Smyslov	2764.92	Y	18	15.92	0.261	15.48	0.179
Ståhlberg	2721.93	N	8	13.10	0.016	14.00	0.041
Szabó	2716.21	N	13	12.89	0.012	13.78	0.031
Taimanov	2713.92	Y	14	14.30	0.046	13.69	0.028
				Soviet win?	0.757		0.485

Notes: 1994 ties are predicted under collusion (78% 2-way, 18% 3-way, 4% 4-way or more), 1950 ties are predicted under no-cartel (80% 2-way, 16% 3-way, 4% 4-way or more).

the Belgrade Candidates 88% of the time. Under the competitive counterfactual, Miguel Najdorf's probability of victory rises from 6% to 12%, and the probability of a Soviet victory falls to 73%.

The 1953 Zurich Candidates (also a double round robin) is depicted in Table 7. The number of participants increased from ten to fifteen in response to criticisms of the limited Western participation in Belgrade 1950 due to visa and political concerns stemming from the start of the Cold War. Hence the successful players from the regular cycle were augmented by the strongest Western players that were felt by FIDE to have been arbitrarily cut off from participating in the previous cycle. By all accounts, Najdorf and Reshevsky were the strongest active players in the West and Euwe of Holland (though now past his prime) was the only living ex-world champion. Retroactive grading and Sonas ratings show that Reshevsky was the favorite going into the Zurich 1953 Candidates Tournament.

Of all our simulations, Zurich shows the greatest difference between the probable winners when going from independent play to collusion. Though legendary for the quality of the games played and the high quality of the top competitors, continued concerns about Soviet collusion and political pressure against the Estonian Soviet Keres have dogged the reputation of this tournament. Our calculations now suggest that the real loser from any collusion was likely to have been the American GM Reshevsky, the strongest American player in the decades prior to the rise of Bobby Fischer and arguably the second strongest American player in the 20th century. Our calculations indicate that he had a 29% chance of winning a fair tournament. With collusion, his chances fell to 15%.[20] After the loss in Zurich, Reshevsky continued to play at a high level but never had a realistic chance of challenging for the world championship again.[21]

Amsterdam 1956 (Table 8) is less interesting because the only non-Soviet in the top group was Oscar Panno who was never among the best half dozen in the world. However, it does illustrate the obvious point that Soviet dominance was not entirely a result of collusive behavior. Most of the world's top players came from the USSR. Simulations indicate that even under our competitive counterfactual, there was a 96% chance of a Soviet advancing in that tournament.

Yugoslavia 1959 (Table 9) saw the debut in the Candidates tournaments of Bobby Fischer, but he was only 16 years old and nowhere near his peak strength. Conversely, the young genius Mikhail Tal from Latvia had enjoyed a meteoric rise to become the leading player in the USSR and then the world. He was a rightful favorite. Moreover, in the case of young players with rapidly increasing ability, it is well established that ratings have a tendency to underestimate their strength, which changes quickly as they approach their peak. Tal won the tournament convincingly and then went on to become world champion by defeating Botvinnik. However, poor health contributed to his losing the title back to Botvinnik in 1961. He never fully overcame his health problems, though he remained one of the chess elite for two more decades.

Curacao 1962 (Table 10) was the famous tournament in which a mature Fischer accused the Soviets of colluding, an accusation supported by Korchnoi after his defection. However, Fischer's ratings suggest that he was not the *ex ante* favorite, even if his rating/performances put him only a few points shy of the leaders. Even if his true rating were higher (because

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[20] It is also possible that Reshevsky might have been less competitive than the Soviets in long tournaments because he was no longer a chess professional. He entered accounting to support his family and could not devote himself exclusively to tournament chess. Nonetheless his performances in the early 1950s were remarkable and Zurich '53 was unquestionably a high point that could have been won had a couple of games simply "gone the other way".

[21] The most extreme claim about collusion at Zurich came in 2002, when Korchnoi claimed that a pact had been arranged to allow Smyslov to win (Plisetsky and Voronkov, 2005, p. 107). This accusation, however, has not been supported by other contemporaries, and Smyslov himself has declined to comment on the matter.

Table 8
Monte Carlo of Amsterdam 1956 Candidates. 10,000 simulations using baseline ordered probit estimates (Table 4-II). 10-Player, double round robin (play each opponent 1× as White and 1× as Black).

Players	Rating	Sov?	ActScore	Cartel		No Cartel	
				E (PredScore)	% win	E (PredScore)	% win
Bronstein	2769.63	Y	9.5	10.50	0.186	10.15	0.183
Filip	2652.84	N	8	6.90	0.001	7.47	0.004
Geller	2719.95	Y	9.5	9.47	0.047	9.01	0.043
Keres	2782.23	Y	10	10.76	0.252	10.45	0.245
Panno	2672.63	N	8	7.33	0.002	7.91	0.010
Petrosian	2755.26	Y	9.5	10.23	0.142	9.84	0.132
Pilnik	2639.45	N	5	6.60	0.001	7.16	0.002
Smyslov	2793.39	Y	11.5	10.98	0.326	10.70	0.323
Spassky	2714.37	Y	9.5	9.35	0.040	8.86	0.039
Szabó	2695.05	N	9.5	7.87	0.005	8.46	0.020
				Soviet win?	0.992		0.965

Notes: 1989 ties are predicted under collusion (83% 2-way, 15% 3-way, 2% 4-way or more), 1833 ties are predicted under no-cartel (82% 2-way, 15% 3-way, 3% 4-way or more).

Fischer had been improving rapidly in the year or two previous to the tournament), Fischer's poor form/performance in the early rounds of Curacao meant that he had no realistic chance of winning. The Monte Carlo simulations do suggest, however, that he would have suffered noticeably from any collusion; his no-collusion probability of winning was 19%, which dropped to 6% with collusion. Note that we assumed that all the Soviets were colluding and did not model the specific claim of Korchnoi that collusion was a private arrangement of Keres, Petrosian, and Geller.

Table 9
Monte Carlo of Yugoslavia 1959 Candidates. 10,000 simulations using baseline ordered probit estimates (Table 4-II). 8-Player, quadruple round robin (play each opponent 2× as White and 2× as Black).

Players	Rating	Sov?	ActScore	Cartel		No Cartel	
				E (PredScore)	% win	E (PredScore)	% win
Benko	2649.69	N	8	10.54	0.000	11.30	0.003
Fischer	2699.83	N	12.5	12.39	0.006	13.17	0.028
Gligoric	2736.03	N	12.5	13.72	0.028	14.52	0.103
Keres	2742.92	Y	18.5	15.59	0.162	14.76	0.141
Olafsson	2669.53	N	10	11.25	0.001	12.03	0.007
Petrosian	2753.12	Y	15.5	15.93	0.214	15.15	0.191
Smyslov	2761.64	Y	15	16.22	0.278	15.46	0.249
Tal	2765.75	Y	20	16.37	0.311	15.62	0.277
				Soviet win?	0.964		0.859

Notes: 1385 ties are predicted under collusion (89% 2-way, 10% 3-way, 1% 4-way or more), 1529 ties are predicted under no-cartel (86% 2-way, 13% 3-way, 1% 4-way or more).

Table 10
Monte Carlo of Curacao 1962 Candidates. 10,000 simulations using baseline ordered probit estimates (Table 4-II). 8-Player, quadruple round robin (play each opponent 2× as White and 2× as Black).

Players	Rating	Sov?	ActScore	Cartel		No Cartel	
				E(PredScore)	% win	E(PredScore)	% win
Benko	2619.689	N	12	8.72	0.000	9.60	0.000
Filip	2672.711	N	7	10.50	0.001	11.42	0.006
Fischer	2760.832	N	14	13.58	0.062	14.54	0.186
Geller	2740.699	Y	17	14.55	0.126	13.90	0.107
Keres	2742.707	Y	17	14.54	0.124	13.88	0.101
Korchnoi	2769.476	Y	13.5	15.37	0.276	14.81	0.236
Petrosian	2782.001	Y	17.5	15.87	0.409	15.39	0.360
Tal[**]	2768.898	Y	7	11.88	0.002	11.46	0.004
				Soviet win?	0.937		0.808

Notes: 1481 ties are predicted under collusion (87% 2-way, 12% 3-way, 1% 4-way or more), 1432 ties are predicted under no-cartel (87% 2-way, 12% 3-way, 1% 4-way or more).

[**] Tal withdrew for health reasons after 21 games (i.e., he missed seven games).

6. Conclusion

No one can know with certainty what sorts of pressures were imposed on Soviet players during the heyday of the Soviet Union, especially in as politically significant a sport as chess.[22] Some such as Korchnoi or Yuri Averbakh, both top Soviet GMs, have testified to the political pressure faced by players to kowtow to the regime, though others have questioned the accuracy of these testimonies or the extent to which games were "fixed" (e.g. Kingston, 2002). Many of the leading players of that period have now passed away, and many important state documents from the era have not yet been declassified. However, it is not hard to see how the elevation of championship chess to a sport that exemplified the superiority of *Homo Sovieticus* would support both formal and informal pressure on players to bend the rules. Even in a non-ideological environment, it is possible that players from the same nation might have been tempted to come to collusive arrangements in important tournaments. Given the highly delicate nature of Soviet relations with the West in the later years of Stalin's rule and in the decade following his death, it should not be surprising that adherence to fair play was not of the utmost importance. Furthermore, a top Soviet player was in a unique position. He was able to travel abroad (a highly restricted and desirable privilege), and he could earn foreign currency should he succeed in foreign tournaments. Moreover, leading players were provided with generous support from the state and enjoyed numerous other privileges not enjoyed by lesser players or by the general run of Soviet professionals.[23]

Thus, the finding that the Soviets were more likely to draw among themselves in critical FIDE tournaments than when playing other masters should not come as a surprise, but our paper has been the first to provide strong statistical evidence in support of this result. More important, we have shown that such collusion clearly benefited the Soviet players and led to performances against the competition in critical tournaments that were noticeably better than would have been predicted on the basis of past performances and on their relative ratings. The likelihood that a Soviet player would have won every single Candidates tournament up to 1963 was less than one out of four under an assumption of no collusion, but was about three out of five when the possibility of draw collusion is factored in.

As many chess writers have observed, Fischer was not a strong enough favorite to be severely harmed by the draw collusion in the notorious Candidates Tournament in Curacao 1962. Nonetheless, all non-Soviet players suffered from the effects of collusion. More significant is that the American GM Sammy Reshevsky seems to have been the greatest victim of collusive behavior. He was the *ex ante* favorite in the Zurich 1953 Candidates' Tournament, and his second place finish in the tournament was strong enough that even small collusive effects might have meant the difference between success and failure. By all accounts, Zurich was Reshevsky's last and perhaps best chance to play a match for the World Chess Championship.

In addition to the relevance of our results for the history of chess, the addition of draws to the possible outcomes of a contest suggests interesting modifications to the standard tournament literature. Almost all of the existing research work in the economics literature has focused on competitions where the scores are assumed to be monotonic in effort (e.g., golf) or where the outcomes of a match are either a win or a loss (e.g., tennis or most team sports). The possibility of a draw outcome and the fact that the choice of strategy might not affect the expected value of a match but might affect the distribution of outcomes (more draws with fewer wins and losses vs. a higher chance of a win with a higher risk of a loss) pose interesting problems from a theoretical and empirical standpoint (cf. Hvide, 2002). Future research can be devoted to exploring the theoretical implications of this modification to the standard model. One can also ask how the possibility of strategies that "keep the draw in hand" might affect the applications of the tournament literature to structuring incentives in strategic competitions or in designing incentive pay packages in managerial hierarchies. Finally, we draw attention to the underexploited data from chess tournaments. The sheer number of games that have been played and recorded (running into several million games) and the quality of the chess ratings system in place suggest the possibility of valuable microeconometric studies of the economics of sports, tournament models, and the effects of rules changes on performance.

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jebo.2009.01.009.

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[22] Soviet émigré and international grandmaster Genna Sosonko tells of a story, that "at one meeting of the Chess Federation, when the behavior of a player who had committed some misdemeanor was being discussed [former military prosecutor and head of the chess section of the USSR] Baturinsky said heatedly: 'During the war we used to shoot such people'" (Sosonko, 2003, p. 101).

[23] Even as late as 1971, long after the era of Stalin, performance in chess was treated with utmost seriousness. After losing to Fischer 6-0 in 1971, GM Mark Taimanov lost his title of "Merited Master of Sports," was kicked off the Soviet national team, was forbidden to play in international tournaments for two years, and was even forbidden from performing his secondary occupation as a professional pianist (Plisetsky and Voronkov, 2005, p. 237).

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